Implicit coordination in 2-agent team problems. Application to distributed power allocation

B. Larrousse, A. Agrawal and S. Lasaulce

Laboratoire des Signaux et Systèmes
CNRS - Supélec - Université Paris Sud

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Outline

Introduction

Distributed optimization problem

- Two agents X_1,X_2, a random state X_0.
- $|\mathcal{X}_i| = n_i < \infty$, $i = 0, 1, 2$.
- Utility over T stages:

$$\overline{u} = \frac{1}{T}\mathbb{E}\left(\sum_{i=1}^{T} u(x_{0,i}, x_{1,i}, x_{2,i})\right)$$

- X_0: i.i.d. random process with fixed probability distribution.
- Observation structure: only X_1 observes (perfectly) X_0 *in advance*.
- No dedicated channel for communication.

Limiting achievable coordination performance ?

→ Optimization problem.

→ Information theoretic constraint.

Definitions

Entropy and Mutual Information

For any $(X, Y) \in (\mathcal{X} \times \mathcal{Y})$ with joint law $q(\cdot, \cdot)$:

- Conditional entropy of X given Y:

$$H_q(X|Y) = -\sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} q(x, y) \log_2 \frac{q(x, y)}{q_Y(y)} \tag{1}$$

  where $q_Y(\cdot)$ marginalization of the joint distribution $q(\cdot, \cdot)$;

- Mutual information between X and Y:

$$I_q(X; Y) = -\sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} q(x, y) \log_2 \frac{q(x, y)}{q_X(x) q_Y(y)} \tag{2}$$

Information Constraint

Particular observation structure gives

Information constraint [Gossner et al. 2006]

$$I_q(X_0; X_2) - H_q(X_1|X_0, X_2) \leq 0 \qquad (3)$$

with $q \in \Delta(\mathcal{X}_0 \times \mathcal{X}_1 \times \mathcal{X}_2)$.

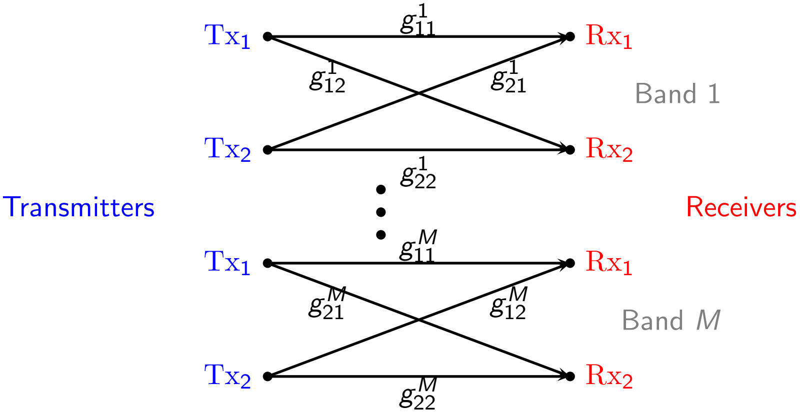
Remark: Generalized to noisy observations in [Larrousse, Lasaulce 2013]

Contribution

Study of the optimization problem:

→ Maximize expected utility with information constraint, marginal constraint and probability constraints.

M-band interference channel



$M \geq 1$ non-overlapping frequency bands.

Discrete power allocation policies and channel gains g_{ij}^m.

Goal: find **best joint distribution(s)**, i.e. best correlation between agent's actions and the random state:

Optimization problem

$$
\begin{aligned}
\min \quad & -\mathbb{E}_q[w] = -\sum_{i=1}^{n_0 n_1 n_2} q_i w_i \\
\text{s.t.} \quad & \sum_{i=1}^{n_0 n_1 n_2} q_i = 1 \\
& \sum_{j=1+(i-1)n_1 n_2}^{i n_1 n_2} q_j = \Pr[X_0 = i], \qquad \forall i \in \{1, \ldots, n_0\} \\
& q_i \geq 0, \qquad \forall i \in \{1, 2, \ldots, n_0 n_1 n_2\} \\
& I_q(X_0; X_2) - H_q(X_1 | X_0, X_2) \leq 0
\end{aligned} \tag{4}
$$

→ Convex optimization problem.

Analytical Results

Proposition

If there exists a permutation such that the payoff vector w can be strictly ordered:

→ *Active* information constraint at optimum.

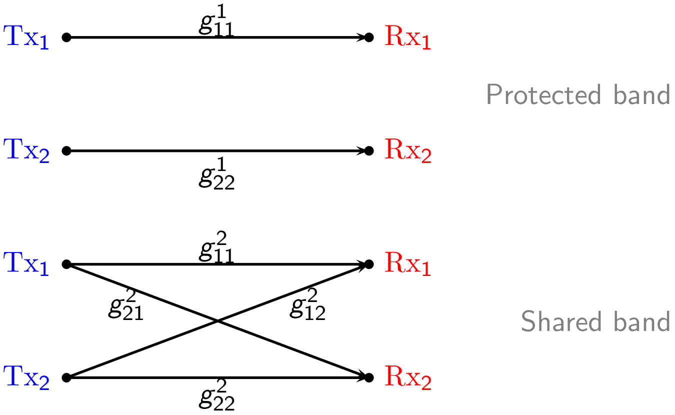
→ Unique solution.

Tools

→ ∃ strictly feasible point → Karush Kuhn Tucker conditions.

→ Uniqueness: convexity and Log-sum inequality [Cover 2006].

Numerical results: 2-Band interference channels[1]



Power sets: $\mathcal{P}_i = P_{\max}\left\{(0,1),(1,0),(\frac{1}{2},\frac{1}{2})\right\}, \quad i \in \{1,2\}$ (5)

[1] [Mochaourab 09]

Parameter values

Two regimes (shared band):

→ High interference regime (HIR):

$$(\pi_{11}^2, \pi_{12}^2, \pi_{21}^2, \pi_{22}^2) = (0.5, 0.1, 0.1, 0.5)$$

→ Low interference regime (LIR):

$$(\pi_{11}^2, \pi_{12}^2, \pi_{21}^2, \pi_{22}^2) = (0.5, 0.9, 0.9, 0.5)$$

Channel gains:

$$g_{ii}^1 \in \{0.1, 1.9\}, \quad i \in \{1, 2\} \tag{6}$$

$$g_{ij}^2 \in \{0.15, 1.85\}, \quad (i, j) \in \{1, 2\}. \tag{7}$$

$$g_{ij}^k \text{ i.i.d. } g_{ij}^k \sim \mathcal{B}(\pi_{ij}^k) \text{ for } k = 1, 2$$

with

$$P(g_{ii}^1 = 0.1) = \pi_{ii}^1 \tag{8}$$

and

$$P(g_{ij}^2 = 0.15) = \pi_{ij}^2. \tag{9}$$

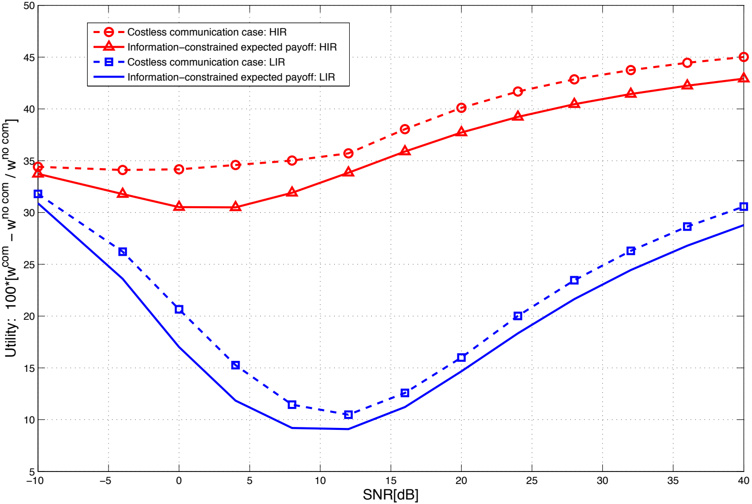
Utility

Utility function:

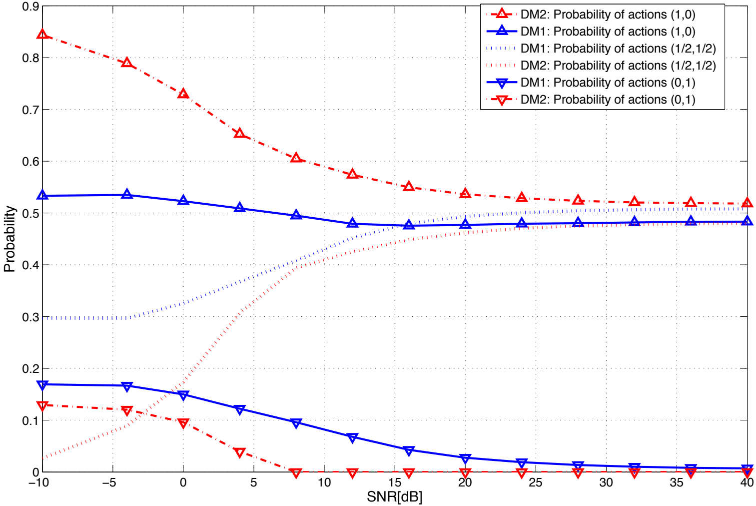
$$u : \left| \begin{array}{ccc} \mathcal{G} \times \mathcal{P}_1 \times \mathcal{P}_2 & \rightarrow & \mathbb{R}^+ \\ (g, p_1, p_2) & \mapsto & \sum_{i=1}^{2} \sum_{m=1}^{2} B_m \log_2 \left(1 + \frac{g_{ii}^m p_i^m}{\sigma^2 + g_{-ii}^m p_{-i}^m} \right) \end{array} \right. \quad (10)$$

Reference: Blind Policy (BP). Transmitters don't know anything about channel gains and $p_1 = p_2 = P_{\max}(\frac{1}{2}, \frac{1}{2})$ at every stage.

Relative gain in terms of expected payoff



Marginal probability distributions $q_{X_1}(\cdot)$ $q_{X_2}(\cdot)$



Conclusion

- → Embedding coordination information into the power allocation levels is highly beneficial.
- → Specific application but approach more general, with high potential.
- → Generalized version (e.g. imperfect monitoring, continuous power allocation) will be provided in future works.

Thank you for your attention.
Do not hesitate to ask questions.
For more: http://benjamin.larrousse.fr/