Coded Power control: Performance Analysis

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Outline

A Coordination problem: Online Matching Pennies [Gossner et al.]

- Two agents, represented by X_1 and X_2, with $\mathcal{X}_1 = \mathcal{X}_2 = \{0,1\}$.
- A random state X_0 i.i.d. $\mathcal{B}(1/2)$, with $\mathcal{X}_0 = \{0,1\}$.
- Stage utility function:

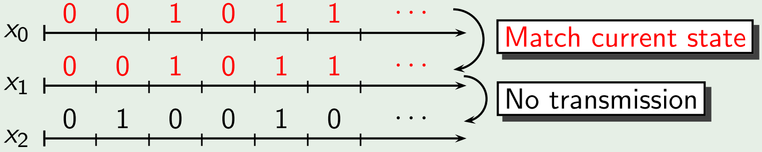
$$w(x_0, x_1, x_2) = \begin{cases} 1 & \text{if } x_0 = x_1 = x_2 \\ 0 & \text{otherwise} \end{cases} .$$

- Particular observation structure: only Agent 1 observes X_0 in advance.
- Scenario repeated T times, $T \in \mathbb{N}$.
- Agents maximize the averaged utility:

$$\underline{w} = \frac{1}{T}\mathbb{E}\left(\sum_{i=1}^{T} w(x_{0,i}, x_{1,i}, x_{2,i})\right)$$

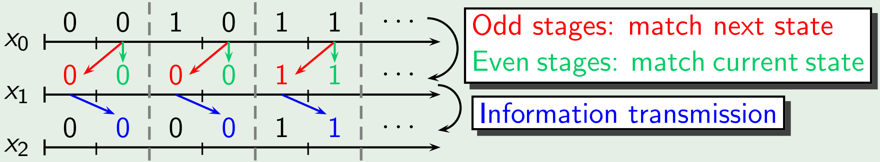
Questions: useful to exchange information ? Maximum utility ?

First Strategy: no communication



→ Expected utility: **0.5**.

Second Strategy: odd/even.



→ Expected utility: **0.625**.

Limiting performance

Result: Online Matching Pennies

- → Maximum utility (with global knowledge): 1.
- → Maximum with an observation structure (i.e., communication constraint): 0.81.

Questions

- Optimal tradeoff between the costs and the benefits of communication ?
- Characterization of this constraint ?

Problem statement

Definition of the distributed optimization problem

- Two agents X_1,X_2, a random state X_0 with fixed law $\rho(x_0)$.
- $|\mathcal{X}_i| < \infty$, $i = 0, 1, 2$.
- Utility over T stages:

$$\underline{w} = \frac{1}{T}\mathbb{E}\left(\sum_{i=1}^{T} w(x_{0,i}, x_{1,i}, x_{2,i})\right)$$

- Y signal on X_1 with fixed law $\Gamma(y|x_1)$.
- Strategy of Agent 1: $(\sigma_{1,i})_{1\leq i\leq T}$ with:

$$\sigma_{1,i} : \mathcal{X}_0^T \times \mathcal{X}_1^{i-1} \times \emptyset \to \mathcal{X}_1$$

- Strategy of Agent 2: $(\sigma_{2,i})_{1\leq i\leq T}$ with:

$$\sigma_{2,i} : \mathcal{X}_0^{i-1} \times \mathcal{Y}^{i-1} \times \mathcal{X}_2^{i-1} \to \mathcal{X}_2$$

Auxiliary notion

Definition (Implementability)

$P_{X_{0,i},X_{1,i},X_{2,i},Y_i}$: joint distribution induced by $(\sigma_{1,i},\sigma_{2,i})_{i\geq 1}$ at stage i.
The distribution $\overline{Q}(x_0,x_1,x_2)$ is implementable if there exists a pair of strategies $(\sigma_{1,i},\sigma_{2,i})_{i\geq 1}$ such that for all (x_0,x_1,x_2),

$$\frac{1}{T}\sum_{i=1}^{T}\sum_{y} P_{X_{0,i},X_{1,i},X_{2,i},Y_i}(x_0,x_1,x_2,y) \rightarrow \overline{Q}(x_0,x_1,x_2)$$

as $T \rightarrow +\infty$.

Feasible utilities

A certain utility value $\underline{w}$ is reachable if and only if there exists an implementable distribution Q such that $\underline{w} = \mathbf{E}_Q[w]$.

Theorem (Characterization of implementable distribution)

Let $\overline{Q} \in \Delta(\mathcal{X}_0 \times \mathcal{X}_1 \times \mathcal{X}_2)$ *with* $\sum_{(x_1,x_2)} \overline{Q}(x_0,x_1,x_2) = \rho(x_0)$. *The distribution* $\overline{Q}$ *is implementable if and only if there exists* $Q \in \Delta(\mathcal{X}_0 \times \mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{Y})$ *which verifies:*

$$I_Q(X_0; X_2) \leq I_Q(X_1; Y|X_0, X_2)$$

where the arguments of the mutual information $I_Q(.)$ *are defined from* Q *and* $Q(x_0,x_1,x_2,y) = \overline{Q}(x_0,x_1,x_2)\Gamma(y|x_1)$.

Remark: This theorem also characterizes expected payoff.

Optimization problem

Our problem is an optimization problem with *convex* constraint:

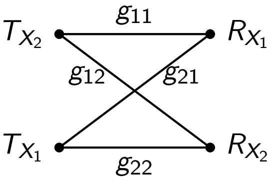
$$
\begin{array}{lrcl}
\text{minimize} & -\mathbb{E}_{\underline{q}}[w] = -\sum_{\ell=1}^{L} q_\ell w_\ell & & \\
\text{subject to} & I_{\underline{q}}(X_0; X_2) - I_{\underline{q}}(X_1; Y|X_0, X_2) & \leq & 0 \\
 & -q_\ell & \leq & 0 \\
 & -1 + \sum_{\ell=1}^{L} q_\ell & = & 0 \\
 & \forall\, x_0, \sum_{\ell \in \mathcal{L}_{X_0}(x_0)} q_\ell - \rho(x_0) & = & 0 \\
 & \forall\, (x_1, y), \dfrac{\sum_{\ell \in \mathcal{L}_{X_1,Y}(x_1,y)} q_\ell}{\sum_{\ell \in \mathcal{L}_{X_1}(x_1)} q_\ell} - \Gamma(y|x_1) & = & 0
\end{array}
$$

It can be solved numerically.

Application: flat-fading interference channel (IC)

Application: power control

- Agents: transmitters.
- Actions: power levels.
- Random state: channel gains.

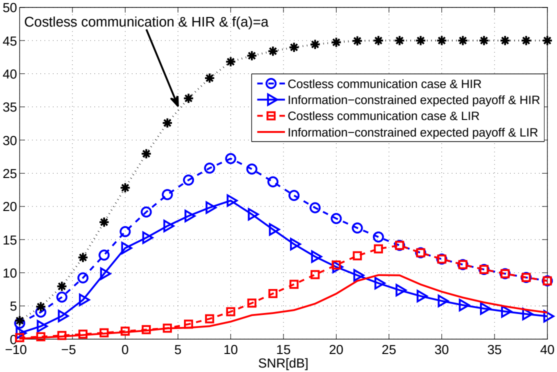


$$\forall\ i,k \in \{1,2\} : g_{ik} \in \{0.1, 1.9\}$$
$$g_{ik} \sim \mathcal{B}(\pi_{ik})$$
$$x_i \in \mathcal{X}_i = \{0, P_{\max}\}$$

- Utility: $w^{\mathrm{IC}} = \sum_{i=1}^{2} \log_2(1 + \mathrm{SINR}_i)$ with $\mathrm{SINR}_i = \frac{g_{ii} x_i}{\sigma^2 + g_{ki} x_k}$.
- Observation: $Y \equiv X_1$.

Coded Power Control policies :

- Semi-coordinated PC (SPC) policy: $x_2 = P_{\max}$, $x_1^\dagger \in \arg\max_{x_1} w^{\mathrm{IC}}(x_0, x_1, P_{\max})$.
- Optimal CPC policy: maximum of the convex optimization problem.
- Costless communication case: maximum of w^{IC} at any stage.



Conclusion

- Shannon theory helps us to find the right information constraint.
- Explicit characterization of feasible utility.
- Methodology used to derive optimal performance is general.

Thank you for your attention.
Do not hesitate to ask questions.
For more: http://benjamin.larrousse.fr/

References

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