Crawford-Sobel meet Lloyd-Max on the grid

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   - Game theoretic model

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   - Signaling scheme

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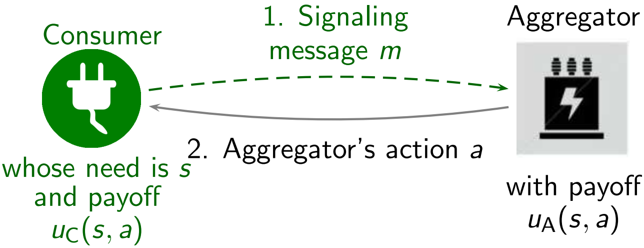
References



- Consumer's objective (operating cost):

$$u_{\mathrm{C}}(s,a) = -(s-a)^2 \tag{1}$$

- Aggregator's objective:

$$u_{\mathrm{A}}(s,a) = -(s-a)^2 - \mathbf{b}e^a \tag{2}$$

- $\mathbf{b} > 0$: **bias** between Consumer's and Aggregator's objectives.

Strategic Information Transmission: Cheap Talk game [2]

- Sender with type $s \in [0,1]$, sends $m \in \{1; 2; \dots; M\}$ with $M < \infty$.
- Receiver chooses an action $a \in [0,1]$.
- Both get a *distinct* payoff depending on s and a, not on messages.
- Payoff functions u_S and u_R verify:

$$\forall i \in \{\mathrm{S}, \mathrm{R}\}, \begin{cases} \forall s, & \exists a, \frac{\partial u_i}{\partial a}(s,a) = 0 \\ \forall (a,s), & \frac{\partial^2 u_i}{\partial a^2}(s,a) < 0 \\ \forall (a,s), & \frac{\partial^2 u_i}{\partial s \partial a}(s,a) > 0 \end{cases} . \quad (3)$$

Solution concept: Nash Equilibrium [3]

Profil of strategies $(\sigma_1^*, \dots, \sigma_n^*) \in \Sigma = (\Sigma_1, \dots, \Sigma_n)$ such that for each i:

$$\forall \sigma_i \in \Sigma_i, \qquad u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i, \sigma_{-i}^*) \quad (4)$$

where $\sigma_{-i} = (\sigma)_{\{1,\dots,n\}\setminus\{i\}}$ and $u_i : \Sigma \to \mathbb{R}$

Non-aligned objectives

Objectives are *non-aligned* if:

$$\arg\max_a u_R(s,a) \neq \arg\max_a u_S(s,a) \qquad \forall s \in [0,1] \tag{5}$$

Nash Equilibrium strategies [2]

If non-aligned objectives:
→ It exists $M_b^\star$ such that, for every $1 \leq M \leq M_b^\star$, there exists at least one equilibrium of size M.
→ Equilibrium strategy for Sender: partition of $[0,1]$ of size M. One message for each interval.
→ Equilibrium strategy for Receiver: M actions, one for each element of a particular interval.

Quantization

- Signal s with PDF $f_S(s)$ mapped to $M < \infty$ representative levels $\hat{s}$.
- Goal: minimize Mean Squared Error: $\mathbb{E}[(s - \hat{s})^2]$.
- Solution: Lloyd-Max algorithm.
  - → Iterative optimization of thresholds and representative levels.
  - → $M - 1$ decision thresholds exactly half-way between representative levels.
  - → M representative levels in the centroid of the PDF between two successive decision thresholds.

Our contribution

- Main difference: two distinct objectives (bias).
  → Non-trivial analysis.
- Results:
  - → Lloyd-Max algorithm with distinct objectives: Best-Response Dynamics.
  - → It exists a *maximum* number $M^\star_{\mathbf{b}}$ of representative levels.

Model

- Aggregator and consumer with *non-aligned* objectives.
- Assumption: law of power need known by both aggregator and consumer.
- Objective: maximize the expected objective functions with respect to beliefs.
- Belief: bayesian estimation of the power need.

→ Application of Cheap talk game model.

Signaling scheme

- Consumer's strategy:

$$f : \left| \begin{array}{ccc} [0,1] & \rightarrow & \{1,2,...,M\} \\ s & \mapsto & m \end{array} \right. . \quad (6)$$

- Aggregator's strategy:

$$g : \left| \begin{array}{ccc} \{1,2,...,M\} & \rightarrow & [0,1] \\ m & \mapsto & a \end{array} \right. . \quad (7)$$

Comments

- Deterministic mappings (due to concavity)
- f: partition of the space of possible power need.
- g: representative of each interval.

→ Quantization [1], with *distinct* objectives.

Link with Lloyd-Max algorithm

- How to find "optimal" strategies?
- Sequential Best-Response (BR) Dynamics: optimizing iteratively f and g converges to an NE.
- Lloyd-max algorithm: exaclty a sequential BR dynamics for *aligned* objectives.

Aggregator's BR (uniform law of power needs)

Given f, the aggregator's BR $a^\star(m)$ to a message m is:

$$a^\star(m) = \left| \begin{array}{ll} \overline{s}_m - W\left(b\frac{e^{\overline{s}_m}}{2}\right) & \text{if} \quad \overline{s}_m > \frac{b}{2} \\ 0 & \text{if} \quad \overline{s}_m \leq \frac{b}{2} \end{array} \right. \tag{8}$$

where $\overline{s}_m = \frac{s_m + s_{m+1}}{2}$ and W is the Lambert W function.

Consumer's BR (uniform law of power needs)

$M_b^\star$: number of optimal partitions. For a given $1 \leq M \leq M_b^\star$, the optimal partition for the consumer can be defined recursively as:

$$\begin{cases} s_0^\star = 0 \\ s_{m+1}^\star = \phi_b(2s_m^\star - \phi_b^{-1}(s_{m-1}^\star + s_m^\star)) - s_m^\star, \\ \qquad\qquad 1 \leq m \leq M-1 \\ s_M^\star = 1 \end{cases} \tag{9}$$

where $\phi_b(x) = 2x + be^x$ and ϕ_b^{-1} its inverse function.

Result

→ BR define an NE for a fixed number of messages between 1 and $M_b^\star$.
→ NE with $M_b^\star$ Pareto dominates ex ante every other NE.

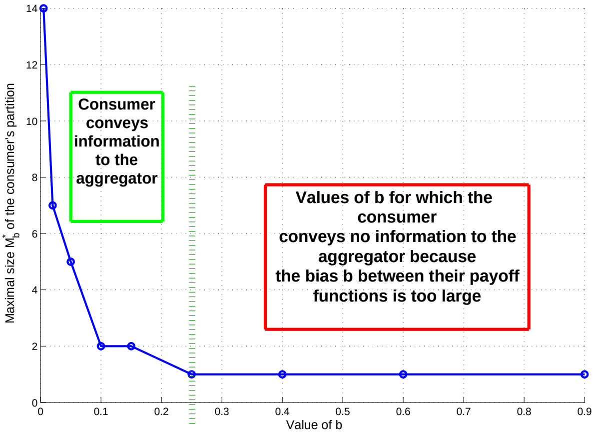
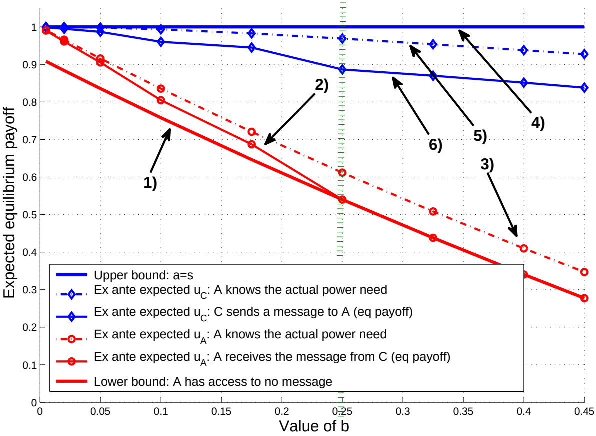


Figure : Maximal number of messages $M_b^\star$ vs the bias b.



1) "A" has access to no message; 2) "A" receives message from "C" (equilibrium payoff); 3) "A" with perfect knowledge of power need. 4) "C" $a = s$; 5) "A" knows the actual power need; 6) "C" sends a message to "A" (equilibrium payoff)

Conclusion

→ Simple signaling problem in Smart Grid resolved with Game Theory.

→ Connections between studied framework and quantization.

→ This opens much more general challenges for source and channel coding with different performance criteria.

Thank you for your attention.
Do not hesitate to ask questions.
For more: http://benjamin.larrousse.fr/

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